Solution Strategies for the Dynamic Warehousing Location under Discrete Transportation Costs

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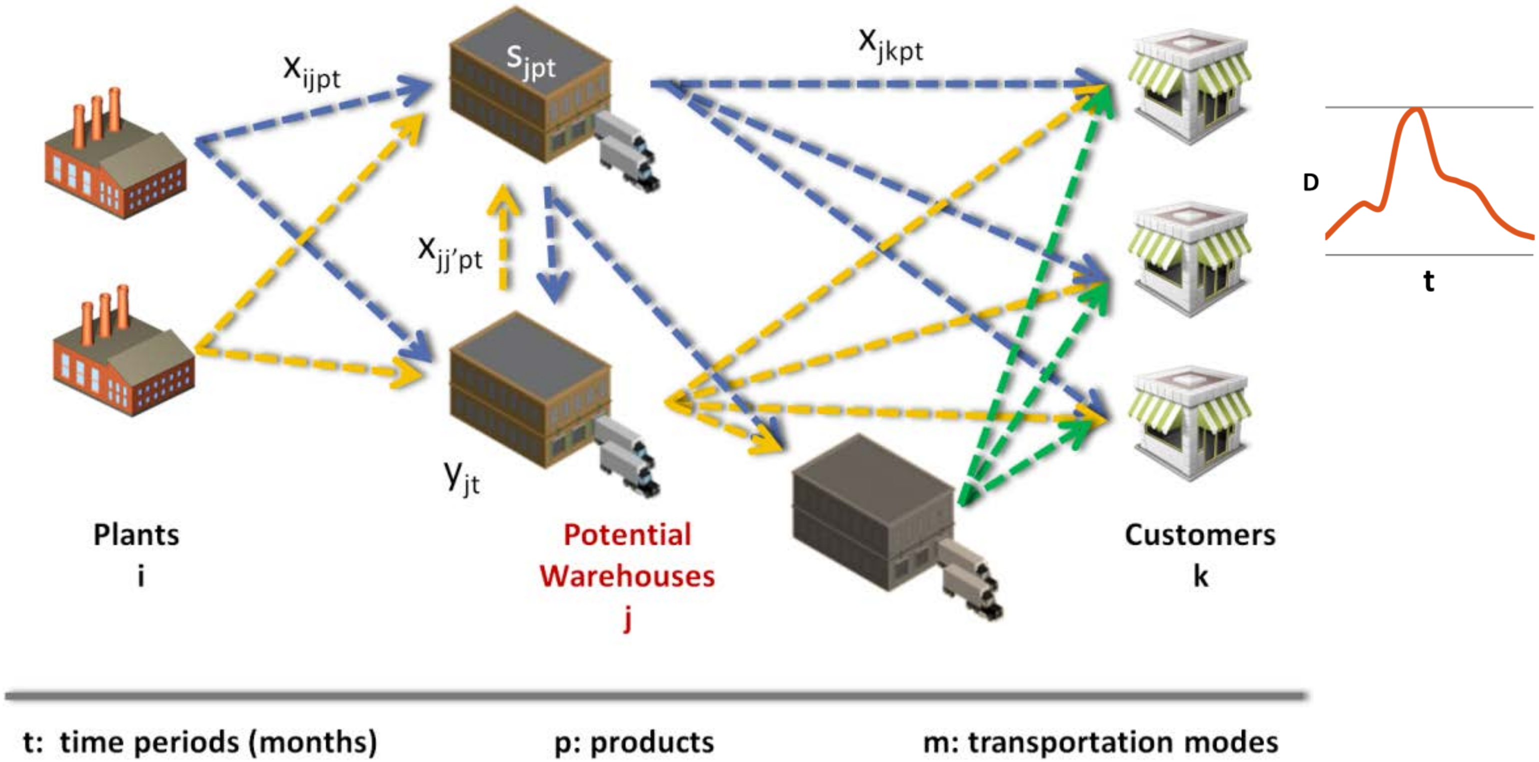
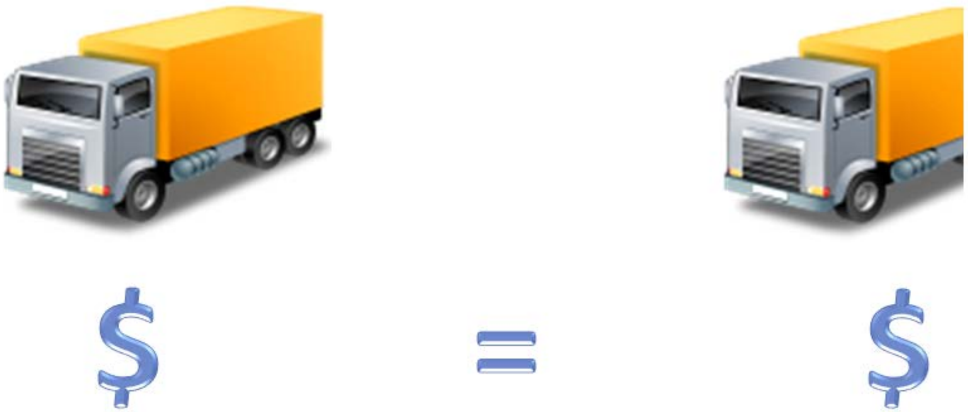
Problem Description



- Decide the number, size, location and contracting length for warehouses
- Dynamic decision of opening/closing warehouses at every period
- Plan the inventory allocation
- Multiple transportation modes, with discrete costs
- 5 year planning horizon
- Seasonal Demand

Discrete Transportation Costs

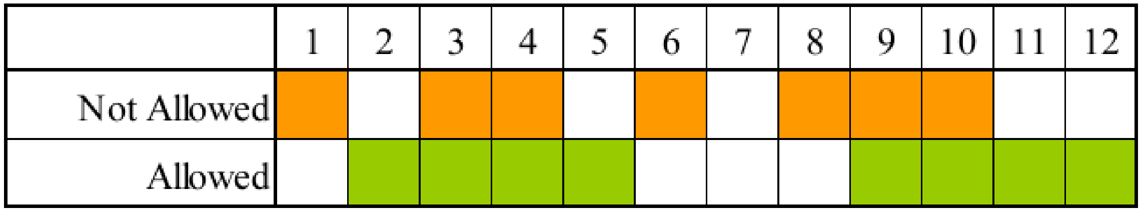


Integer Units

$$\sum_{p} x_{jkpt} \leq \sum_{m} TCap_m \, u_{jkmt}$$

u_{jkmt} integer

Dynamic Contracting Policies



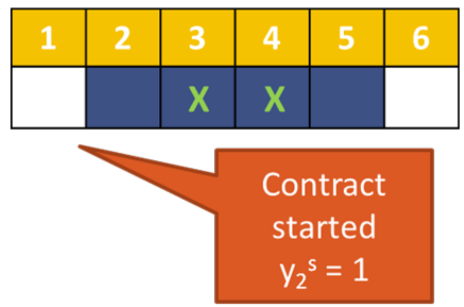
Ex: min wait = 3, min length = 3

1. y_t^s: 1 if a contract is started in period t

$$-y_t + y_{t-1} + y_t^s \geq 0$$

2. y_t^f: 1 if a contract is finished in period t

$$-y_t + y_{t+1} + y_t^f \geq 0$$



Minimum wait

$$\sum_{\tau=t+1}^{t+W} y_\tau + W y_t^f \leq W$$

If a contract is finished in period t the warehouse can not be used in the next W periods

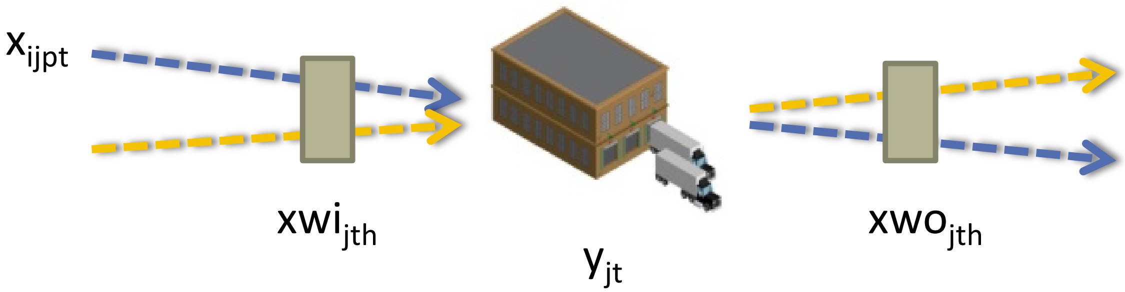
Minimum length

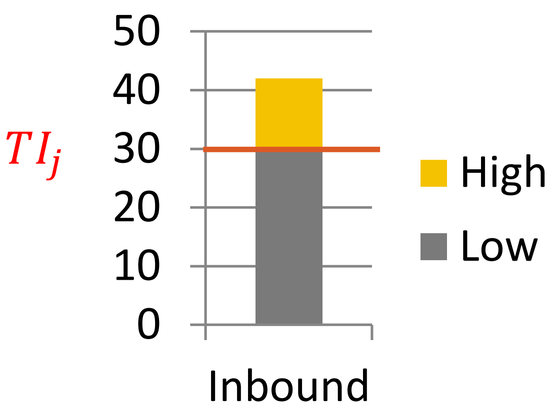
$$\sum_{\tau=t}^{t+L-1} y_\tau \geq L y_t^s$$

If a contract is started in period t the warehouse must be used in the next L periods

Flow Cost Structure

- Inbound and outbound unit cost with penalty for high volume





$$\sum_p \sum_i x_{ijpt} = xwi_{jt,low} + xwi_{jt,high}$$

$$xwi_{jt,low} \leq TI_j$$

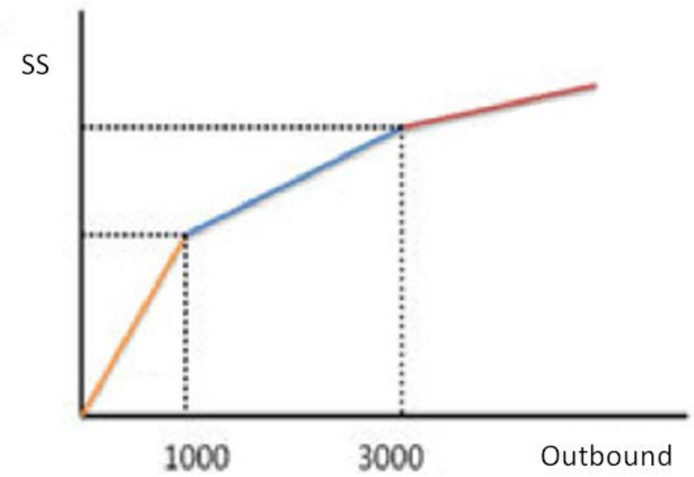
4. Safety Stock with Risk Pooling effect

Safety Stock

- $safety\ stock = z\,\sigma\sqrt{L} \quad \approx \quad ss = \beta x$

Risk Pooling

- Stock out risk decreases with number of customers served

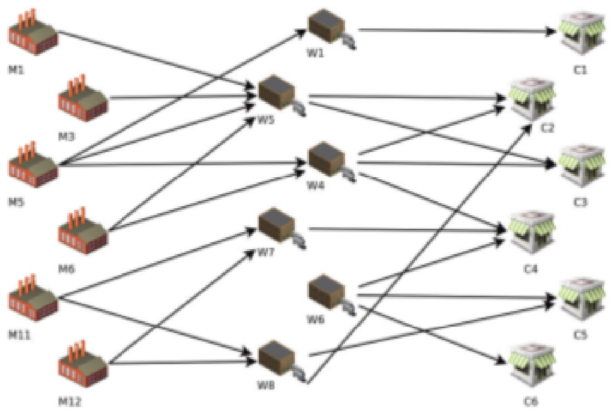
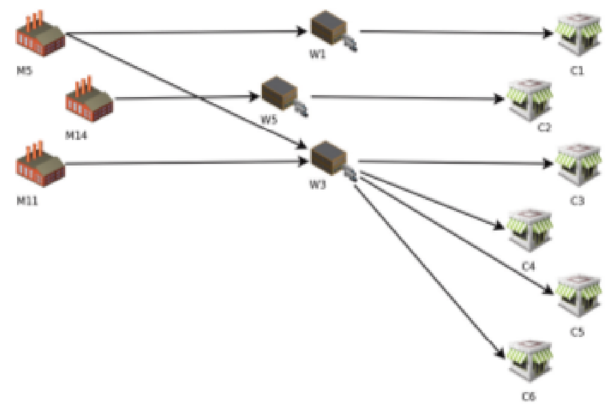


$$\sum_k x_{jkpt} = \sum_n \lambda_{jptn}\, SS_{xn} MY\, D_p$$

$$ss_{jpt} = \sum_n \lambda_{jptn} SS_{xn} SS_{yn}\, MY\, D_p$$

$$\sum_n \lambda_{jptn} = 1$$

Effect of Transportation Costs

Continous Costs	Discrete Costs
	

Continous Costs				Discrete Costs
$83 k	↓	Cost	↑	$262 k
90 s	↓	Solution Time	↑	5 h
193 ton	↓	Inventory	↑	607 ton
807	↑	Shipments	↓	417
1.07 ton/shipment	↓	Shipment Size	↑	2.07 ton/shipment

It is important to consider discrete transportation costs

Tightening Constraints

1. Plants must supply at least enough to meet demand

$$\sum_{i \in I_p} \sum_j x_{ijpt} \geq \sum_k D_{kpt} - \sum_j (s_{jpt} + ss_{jpt}) \qquad \forall p, t$$

2. It is not possible to ship to/from a closed warehouse

$$\sum_i \sum_p x_{ijpt} \leq UB\ y_{jt} \qquad \forall j, t$$

3. No transportation units are used in a lane with a closed warehouse

$$u_{jktm} \leq UB\ y_{jt} \qquad \forall j, k, t, m$$

4. Transportation units for a defined MOT are at most the exclusive mode number of units

$$u_{ijrm} \leq \frac{1}{TCap_m} \sum_{p \in P_i} x_{ijpt} + 1 \qquad \forall i, j, m$$

15% time reduction

Model Reformulations

Motivation:

1. Network design is driven by warehouse-customer transportation costs
2. Customers value consistency in their service
3. Reduce model complexity

Results:

	Original	JKP	JK
CPU (s)	364	60	26
Objective Value	$ 3.29 M	$ 3,32 M	$ 3,35 M
Objective var	-	+ 0.7%	+ 1.5%

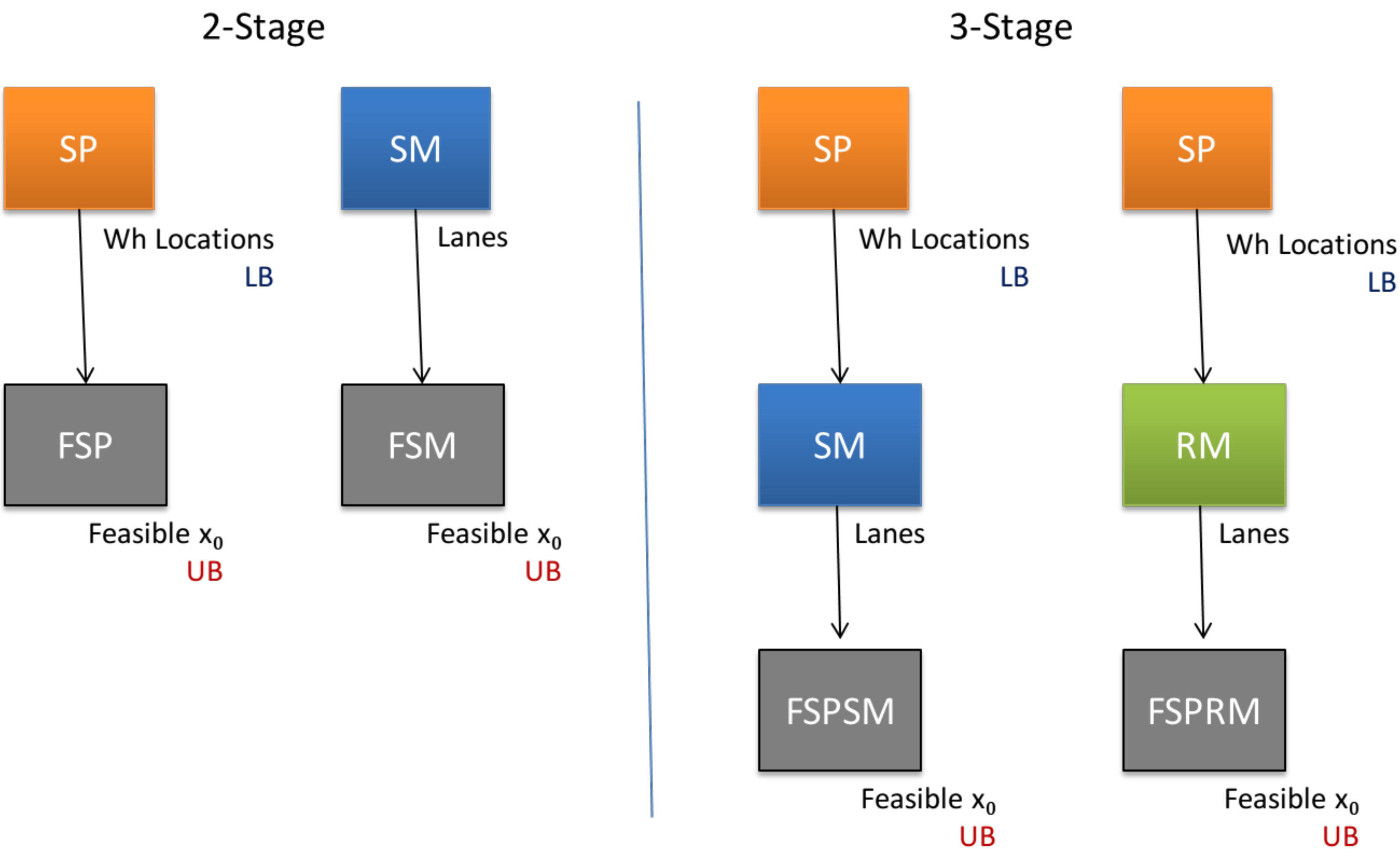
JKP : A customer receives **each product** from a specific warehouse

JK : A customer receives **all products** from a exclusive warehouse

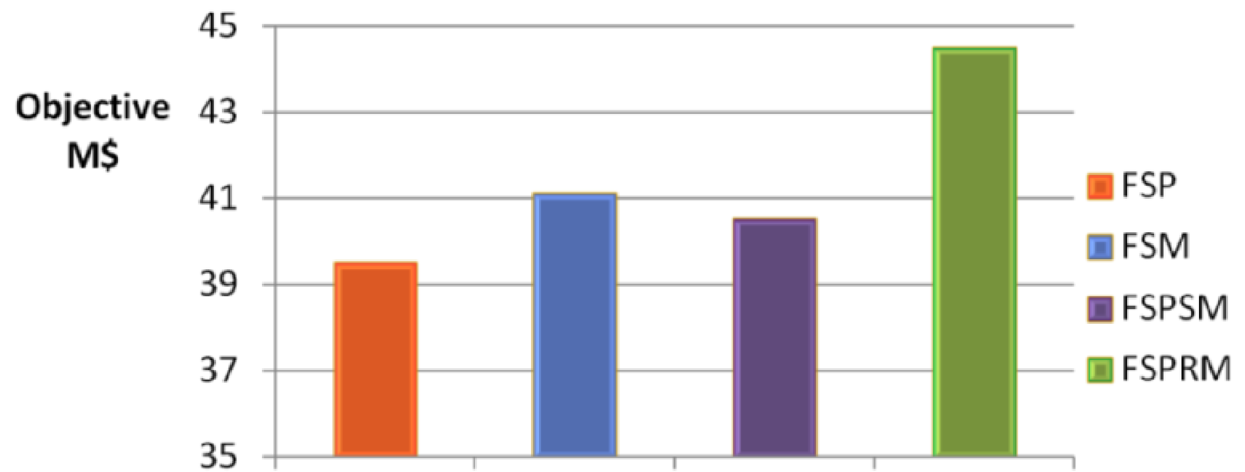
15 Plants, 15 Warehouses, 10 Customers, 10 Products, 12 Months, 16 Modes of Transportation

Multistage heuristics

- SP: All products aggregated as one
- SM: Single transportation mode using the average cost per lane
- RM: Relaxed integrality on transportation units
- F#: Full model with warehouses and/or lanes fixed



Heuristics Results



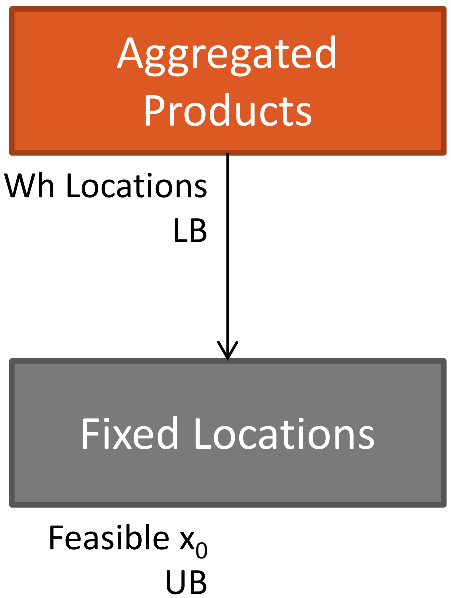
	FSP	FSM	FSPSM	FSPRM
Solution time	10h	21 min	28 min	1h
Proven Optimality Gap	2.8 %	6.8 %	5.5 %	14 %

15 plants, 18 warehouses, 105 customers
15 modes, 78 products, 60 months

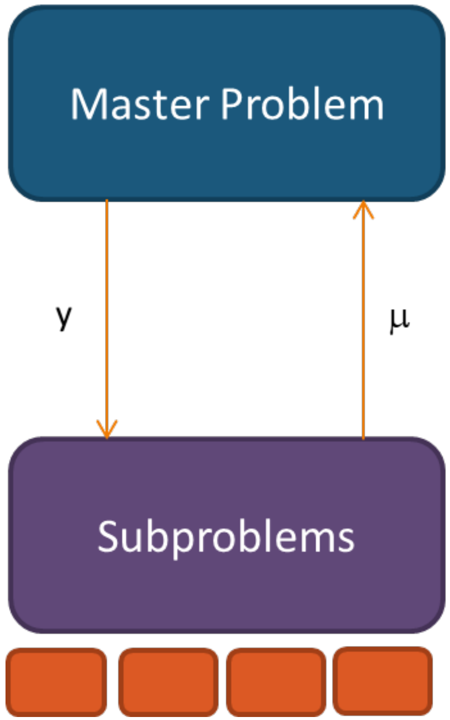
CPLEX 12.6.3
1,000,000 constraints, 1,700,000 variables
281,000 binaries, 235,000 integer

Future Challenge

FSP

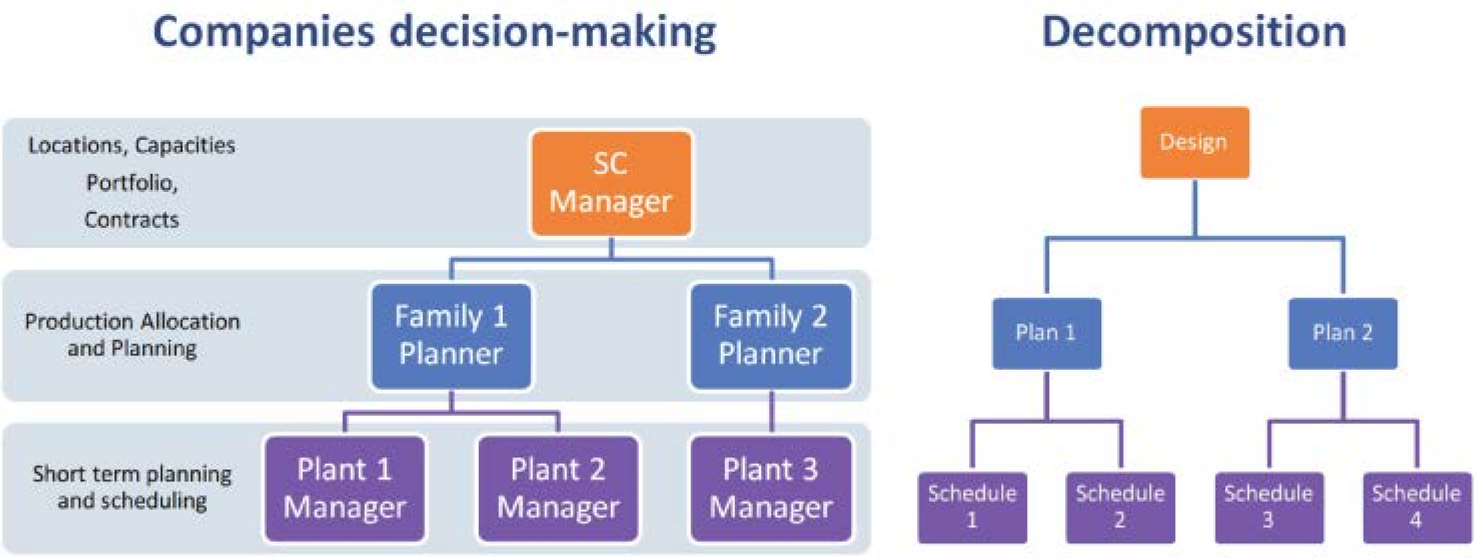


Benders



The heuristics can be upgraded to rigorous decomposition algorithms if valid cuts are obtained from mixed-integer subproblems

Long-Term Goal



Decomposition can mimic the way companies make decisions